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Calculating Mortgage Loans

By Wayne E. Etter **Reprinted June 1995**

Instructor's Notebook presents a lecture on a basic real estate subject. Written by an expert, *Instructor's Notebook* takes readers into the classroom to hear the professor's talk. This regular feature is designed as an introductory lecture on a different topic each issue.

Mortgage loan calculations are based on present value concepts. Although they usually are made with a calculator or a computer, learning how present value basics can be used to calculate the payment provides the understanding needed to solve practical mortgage calculation problems. For a review of present value basic see "Instructor's Notebook," January 1990.

Present Value Basics

A borrower obtains a \$100,000, 10 percent, 25-year loan. Repayment of this loan requires 25 annual payments of \$11,017. Because the annual payments are equal, they are an annuity, and its present value can be calculated. Using a discount rate of 10 percent (and ignoring rounding error), the present value of the annuity is equal to the amount of the loan.

$$\text{Annual payment} \times \underset{(10\%,\ 25\text{ years})}{\text{Annuity factor}} = \underset{\text{of payments}}{\text{Present value}}$$

$$\$11{,}017 \times 9.077 = \$100{,}000$$

* The annuity factor can be calculated by solving for the present value of a \$1-per-year payment, discounted at 10 percent for 25 years.

These terms can be rearranged to calculate the loan payment:

$$\text{Loan amount} \times \frac{1}{\text{Annuity factor}} = \text{Annual payment}$$

$$\$100{,}000 \times \frac{1}{9.077} = \$11{,}017$$

Usually, however, the appropriate mortgage constant is used to calculate the payment. Mortgage constant tables are found in many real estate textbooks and are published in special books of financial tables. The mortgage constant can be calculated by solving for the payment of a \$1 loan using the appropriate interest rate and repayment term.

$$\text{Loan amount} \times \underset{(10\%,\ 25\text{ years})}{\text{Mortgage constant}} = \text{Annual payment}$$

$$\$100{,}000 \times .11017 = \$11{,}017$$

Examination of these two methods indicates that the annuity factor and the mortgage constant are reciprocals:

$$\frac{1}{\text{Annuity factor}} = \text{Mortgage constant}$$

When monthly mortgage payments are required, monthly mortgage constants rather than annual mortgage constants are used. Although most mortgage loans are repaid monthly, annual mortgage loan payments normally are used for illustration.

If the borrower actually receives \$100,000 from the lender and then pays the \$11,017 annually to the lender, the lender earns and the borrower pays a true rate of 10 percent on the loan. In fact, 10 percent is the *internal rate of return* because 10 percent is the rate of discount that makes the present value of the loan payments equal to the loan amount. If less than \$100,000 is received by the borrower but payments of \$11,017 are still made, then the true rate is greater than 10 percent. This situation occurs when points or other financing costs are paid by the borrower to the lender.

The calculated mortgage payment is sufficient to repay the principal amount borrowed during the term of the loan. In addition, the lender receives and the borrower pays the stated rate of interest on the outstanding balance of the loan. Figure 1 illustrates how the principal is repaid during the term of the loan. Note that a small balance remains after the last payment. This will almost always be true because all payments are rounded to the nearest cent.

Sometimes there is a need to estimate a mortgage's unpaid balance as of a certain date. For example, assume a \$50,000 loan was made for 25 years at an 8 percent rate. The appropriate mortgage constant is 0.0937 and the annual payment is \$4,685.

$$\text{Loan amount} \times \text{Mortgage constant} = \text{Annual payment}$$

$$\$50{,}000 \times .0937 = \$4{,}685$$

What is the unpaid balance of the loan after seven payments have been made? Consider that the loan is still an annuity but has only 18 remaining annual payments. To find the unpaid balance, the present value of the annuity is calculated using the annuity factor for 8 percent and 18 years.

$$\text{Annual payment} \times \text{Annuity factor} = \text{Unpaid balance}$$
$$\$4{,}685 \times 9.372 = \$43{,}907.82$$

What if the owner desires to sell the mortgage after receiving the seventh payment? Assume the current mortgage rate for this type of loan is 12 percent. Again, the mortgage is an annuity with 18 remaining payments of $4,685. To determine the current market price of the mortgage, the payments are discounted using the annuity factor for 12 percent and 18 years.

$$\text{Annual payment} \times \text{Annuity factor} = \text{Market value}$$
$$\$4{,}685 \times 7.250 = \$33{,}966.25$$

The decreased value of the mortgage results from the fact that the fixed payments must provide a higher rate of return to the owner of the mortgage than is provided in the mortgage, i.e., 12 percent versus 8 percent. Note that the rate of return and value vary inversely for a security that provides a fixed income stream.

Figure 1. Illustration of a Declining-Balance Mortgage Loan

Loan amount:	$100,000
Interest rate:	11%
Term:	10 years
Mortgage constant:	0.1698014
Payment:	$16,980.14

Year	Payment	Interest	Principal	Balance
1	$16,980.14	$11,000.00	$ 5,980.14	$94,019.86
2	16,980.14	10,342.18	6,637.96	87,381.90
3	16,980.14	9,612.01	7,368.13	80,013.77
4	16,980.14	8,801.52	8,178.62	71,835.15
5	16,980.14	7,901.87	9,078.27	62,756.88
6	16,980.14	6,903.26	10,076.88	52,679.99
7	16,980.14	5,794.80	11,185.34	41,494.65
8	16,980.14	4,564.41	12,415.73	29,078.92
9	16,980.14	3,198.68	13,781.46	15,297.46
10	16,980.14	1,682.72	15,297.42	0.05

Mortgage Constant

From the perspective just presented, mortgage constants are simply numbers from a table used to calculate mortgage loan payments. Although this is true, mortgage constants also indicate the cash cost of borrowing money in much the same way as the interest rate for other types of loans. For example, on a $100,000, 12 percent, interest-only loan, the borrower expects to pay $12,000 annual interest.

$$\frac{\text{Annual interest}}{\text{Loan amount}} = \text{Interest rate}$$

$$\frac{\$12{,}000}{\$100{,}000} = 12 \text{ percent}$$

Because the mortgage loan payment includes principal and interest, the annual payment must be larger than the amount sufficient to pay the annual interest. If a 12 percent, $100,000 mortgage loan is to be repaid in 25 years, the annual payment is $12,750.

$$\text{Mortgage constant} \times \text{Loan amount} = \text{Loan payment}$$
$$.1275 \times \$100{,}000 = \$12{,}750$$

Rearranging terms:

$$\frac{\text{Loan payment (principal+ interest)}}{\text{Loan amount}} = \text{Mortgage constant}$$

$$\frac{\$12{,}750}{\$100{,}000} = .1275 \text{ percent}$$

Thus, the mortgage constant, like the interest rate, expresses the cash cost of borrowing money. Because of this, the mortgage constant is often quoted as an indicator of borrowing costs in a manner similar to the interest rate. When this is done, the mortgage constant is expressed in percentage terms, not as a decimal fraction–12.75 percent, not 0.1275.

The mortgage constant at a particular interest rate always exceeds that interest rate because it includes the amount necessary to repay the loan over the life of the loan and pay the interest on the loan. Therefore, extending the maturity of the loan reduces the payment because the principal repayment is spread over more years. Thus, the mortgage constant is a function of the interest rate and the maturity. A review of a mortgage constant

Figure 2. Reduction in Annual Payments Required to Amortize a $1,000 Loan at Selected Rates and Maturities

Interest Rate (%)	Annual payment 15 years	Annual payment 20 years	Decrease from 15-year payment (%)
6	$102.96	$ 87.19	15.32
8	116.83	101.85	12.82
10	131.47	117.46	10.66
12	146.83	133.88	8.82
Interest Rate (%)	**Annual payment 20 years**	**Annual payment 25 years**	**Decrease from 20-year payment (%)**
6	$ 87.19	$ 78.23	10.27
8	101.85	93.68	8.02
10	117.46	110.17	6.21
12	133.88	127.50	4.76
Interest Rate (%)	**Annual payment 25 years**	**Annual payment 30 years**	**Decrease from 25-year payment (%)**
6	$ 78.23	$ 72.65	7.13
8	93.68	88.83	5.18
10	110.17	106.08	3.71
12	127.50	124.14	2.63

table reveals that as the loan maturity is increased, the mortgage constant decreases from one plus the interest rate to a value almost equal to the interest rate (the interest rate is the mathematical limit of the function).

For example, mortgage payments were calculated at four different interest rates and for increasing maturities. These comparisons are presented in Figure 2. The mortgage constant and, therefore, the annual cost of borrowing decreases as the maturity of the loan increases. Note, however, that **the percentage decrease is largest when the loan term is increased from 15 to 20 years and the decrease is least when the loan term is increased from 25 to 30 years.** Further note that at higher interest rates there is less to be gained in the way of mortgage payment reduction from extending the loan's maturity than there is at lower interest rates.

The ready availability of calculators and computers allows most mortgage loan calculations to be made quickly and easily. Sometimes, however, solving a problem requires more than knowledge of the proper calculator keystrokes. In such cases, basic present value concepts such as annuities and mortgage constants can be used to assist in problem solving. Knowledge of these concepts provides greater understanding of the usefulness of the result.

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